QUANTUM CIRCUITS FOR QUANTUM WALKS ON THE HYPERCUBE

Adina BĂRÎLĂ

InfoTech 2015 - Bulgaria

Quantum computing

- Quantum computing is a field of science which investigates the computational power of computers based on quantum mechanical principles.
- Recent research has proved the potential of quantum computing systems to solve problems that are considered unsolvable due to the necessary computing effort.

Qubit

- The fundamental unit of quantum information is called quantum bit or qubit.
- A qubit can be $|0\rangle$ or $|1\rangle$ (basis states) or a superposition:

$$|\Psi\rangle = \alpha|0\rangle + \beta|1\rangle \qquad (1)$$

where

$$|\alpha|^2 + |\beta|^2 = 1 \qquad (2)$$

If a measure is performed, it obtains

- 0 with probability $|\alpha|^2$ (the state of the qubit becomes $|0\rangle$)
- 1 with probability $|\beta|^2$ (the state of the qubit becomes $|1\rangle$)

Quantum register

- A collection of *n* qubits is called a quantum register of size *n*.
- The state of the quantum register is :

$$|\psi\rangle = \sum_{k=0}^{2^n-1} C_k |k\rangle \qquad (3)$$

where

$$|k\rangle = |k_{n-1}\rangle \ldots |k_1\rangle |k_0\rangle \qquad (4)$$

$$\sum_{k=0}^{2^n-1} |C_k|^2 = 1 \qquad (5)$$

If a measure is performed it obtains $|k\rangle$ with probability $|C_k|^2$ and the state of the system becomes $|k\rangle$.

Quantum gates

- Evolution of a quantum system can be described by a unitary transformation U called gate, in analogy to classical logic gates. Unlike the logic gates, a quantum gate has the same number of inputs and outputs.

- A one-qubit elementary gate is described by a 2x2 matrix:

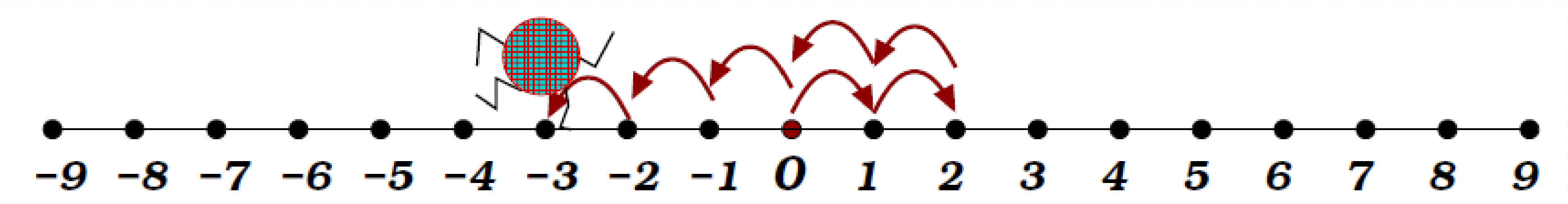
$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

which transforms $|0\rangle$ into a$|0\rangle$+c$|1\rangle$ and $|1\rangle$ into b$|0\rangle$+d$|1\rangle$.

Quantum gates

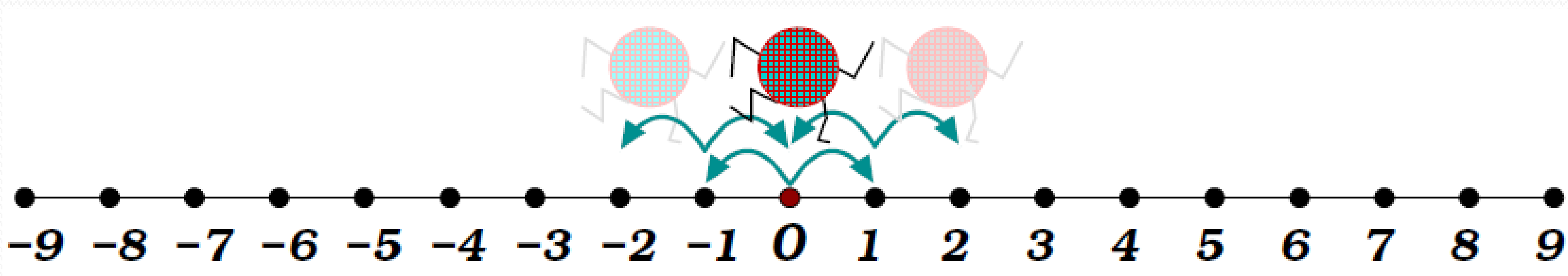
- Hadamard H $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$ - one-qubit
- Controlled-NOT (CNOT) $CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$ 2-qubit gate

Classical random walk



A walker is placed at the origin (0) of a line numbered from –N to N. The walker tosses an unbiased coin and moves either left or right by one position depending on outcome.

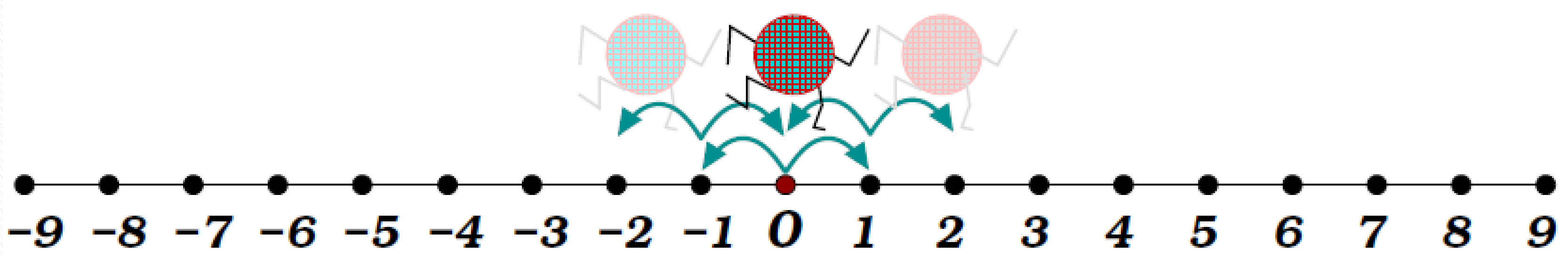
Discrete time quantum walk on the line



- Since quantum operations must be reversible , „toss" must be performed by a unitary operator called *coin operator.*

- If one obtains "head" after tossing, the walker "moves" from the position described by the vector $|n\rangle$ to the position described by $|n+1\rangle$. If one obtains "tail" the walker "moves" from $|n\rangle$ to the position described by $|n-1\rangle$.

Discrete time quantum walk on the line



- the position of the walker is a vector in a Hilbert space H_P with the following computational basis

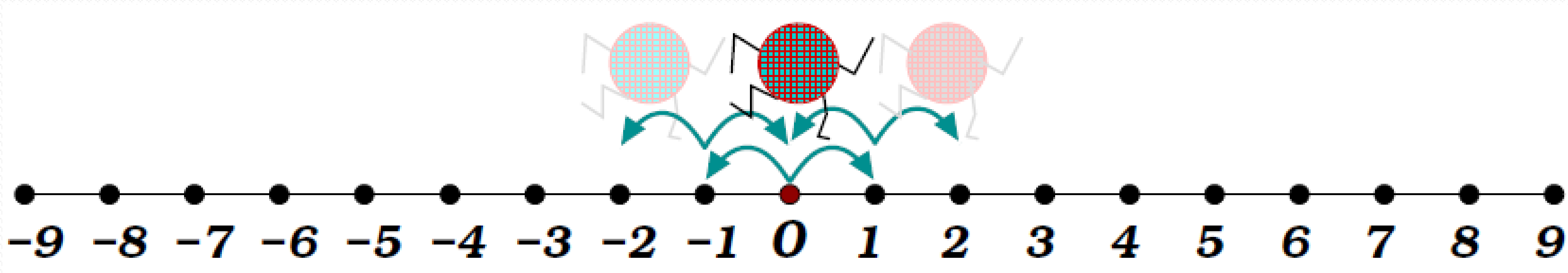
$$\{ |x\rangle : x \in Z \}$$

- The quantum "coin" is a vector in a Hilbert space H_C, with the following computational basis

$$\{|0\rangle, |1\rangle\}$$

- The Hilbert space of the quantum system is $H = H_P \otimes H_C$.

Discrete time quantum walk on the line



The most used coin for unidimensional quantum walks is the Hadamard operator:

$$H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

$$\mathbf{H|0,}\boldsymbol{x}\rangle = \frac{1}{\sqrt{2}}(|0,\boldsymbol{x}\rangle + |1,\boldsymbol{x}\rangle)$$

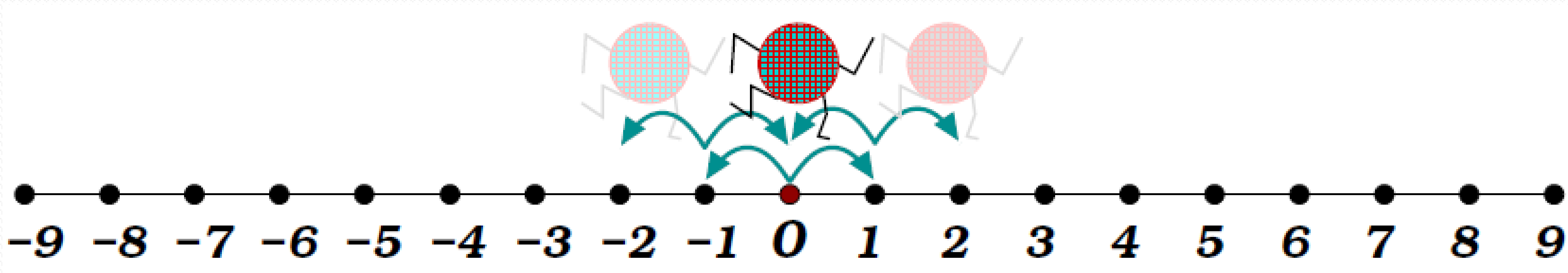
$$\mathbf{H|1,}\boldsymbol{x}\rangle = \frac{1}{\sqrt{2}}(|0,\boldsymbol{x}\rangle - |1,\boldsymbol{x}\rangle)$$

The shift from $|n\rangle$ to $|n+1\rangle$ or $|n-1\rangle$ is described by a unitary operator, called *shift operator* S. This acts as follows:

$$S|0\rangle|n\rangle = |0\rangle|n+1\rangle$$

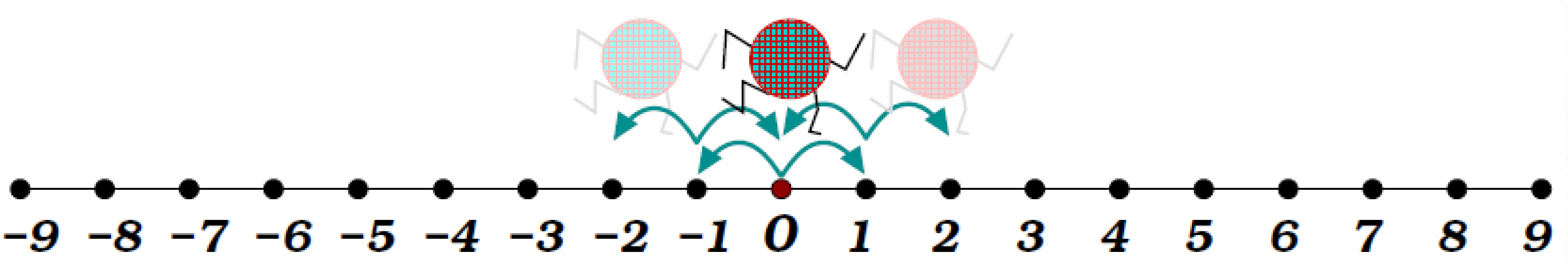
$$S|1\rangle|n\rangle = |1\rangle|n-1\rangle$$

Discrete time quantum walk on the line



In the classical random walk, the walker can only go in one direction at a time. In contrast, in quantum walk he can go in both directions until the measuring operation is performed.

Discrete time quantum walk on the line



The algorithm which implements the quantum walk can be implemented as follows:

1.initialize the system

2.for every iteration

- toss the coin
- shift the position

3.perform measurement

Discret time quantum walk on a regular graph

- The difference between the quantum walk on a regular graph and the quantum walk on the line is the Hilbert space of the quantum system.
- Let G = (V,E) be a regular undirected graph, where V = {1,2,…N} is the set of vertices and E is the set of edges. The Hilbert space of the quantum system is

$$H = H_C \otimes H_v$$

where H_v is the vertices space which has the following computational basis

$$H_v = \{ |v\rangle : v \in Z_N \}$$

where N is the number of vertices, and H_C is the coin space and has the computational basis

$$H_C = \{ |k\rangle : k \in Z_d \}$$

where d is the degree of every vertex.

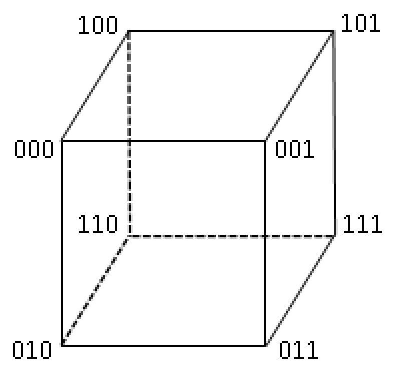
Discret time quantum walk on a regular graph

The walker can move in any of *d* directions.

The shift operator S maps the state $|k,v\rangle$ into $|k,v_j\rangle$, where the (v, v_j) is the *j*-th edge which connects the vertices v *and* v_j.

Discret time quantum walk on the hypercube

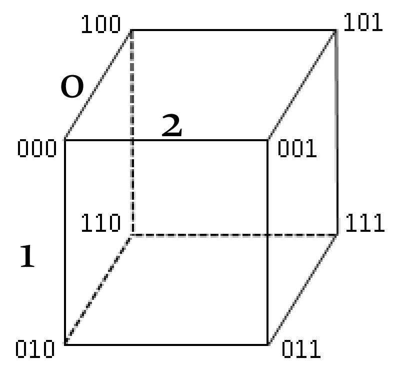
- The hypercube of dimension n is a regular graph with $N = 2^n$ vertices.
- The every vertex degree is n.
- Vertices are labeled by n-bit strings and two vertices are adjacent if and only if their labels differ only by one bit.
- If two vertices differ by the j-th bit, the label of the edge connecting these vertices is j.



Discret time quantum walk on the hypercube

On a hypercube, the shift operator maps the state $|k,v\rangle$ into $|k,v_j\rangle$, where the n-bit strings v and v_j differ by the j-th bit. So, the shift operator S can be represented as follows

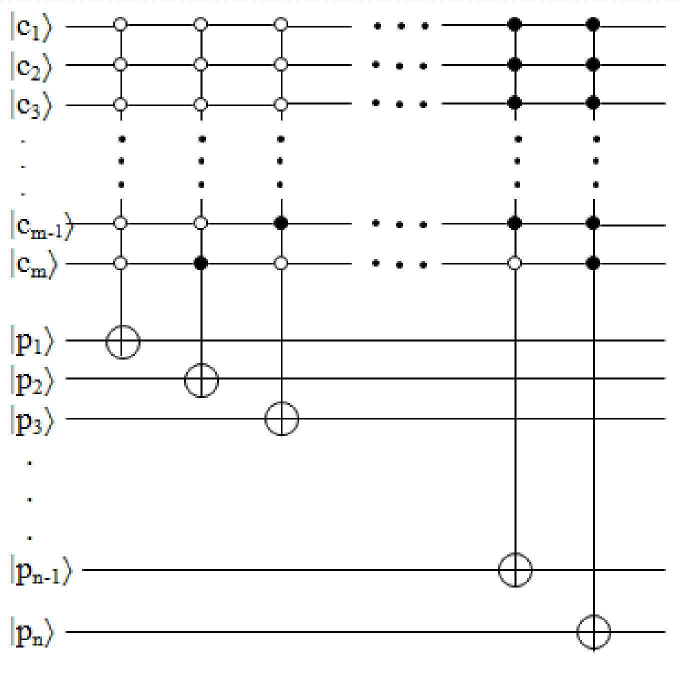
$$S|1\rangle|p_1,p_2,\ldots,p_n\rangle = |1\rangle|p_1\oplus 1, p_2,\ldots,p_n\rangle$$
$$S|2\rangle\,|p_1,p_2,\ldots,p_n\rangle = |2\rangle\,|p_1, p_2\oplus 1,\ldots,p_n\rangle$$
$$\ldots$$
$$S|n\rangle|p_1,p_2,\ldots,p_n\rangle = |n\rangle|p_1,p_2,\ldots,p_n\oplus 1\rangle$$



where $\oplus$ denotes addition modulo two.

Discret time quantum walk on the hypercube

If *n* is a power of two, the action of this operator can be reproduced by (Controlled)^m-NOT quantum gates as shown in figure

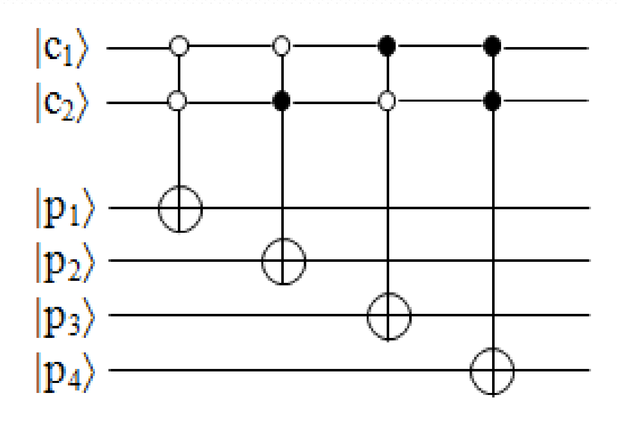


where $|c_1\rangle|c_2\rangle\ldots|c_m\rangle$ is the coin state and $|p_1\rangle|p_2\rangle\ldots|p_n\rangle$ the position state (the vertex state).

Discret time quantum walk on the hypercube

```
procedure walk4(int steps) {
  qureg c[2]; //coin register
  qureg v[4]; //vertices register
  int i;
  int m1;
  int m2;
  for i=1 to steps {
    H(c); //Hadamard coin
    //the first gate
    Not(c);
    CCNot(c[0],c[1],v[3]);
    Not(c);
    //the second gate
    Not(c[1]);
    CCNot(c[0],c[1],v[2]);
    Not(c[1]);
    //the third gate
    Not(c[0]);
    CCNot(c[0],c[1],v[1]);
   Not(c[0]);
    //the last CCNOT gate
    CCNot(c[0],c[1],v[0]);
  }
  measure c,m1;
  measure v,m2;
  print "m1 = ",m1, " m2= ", m2;
}
```

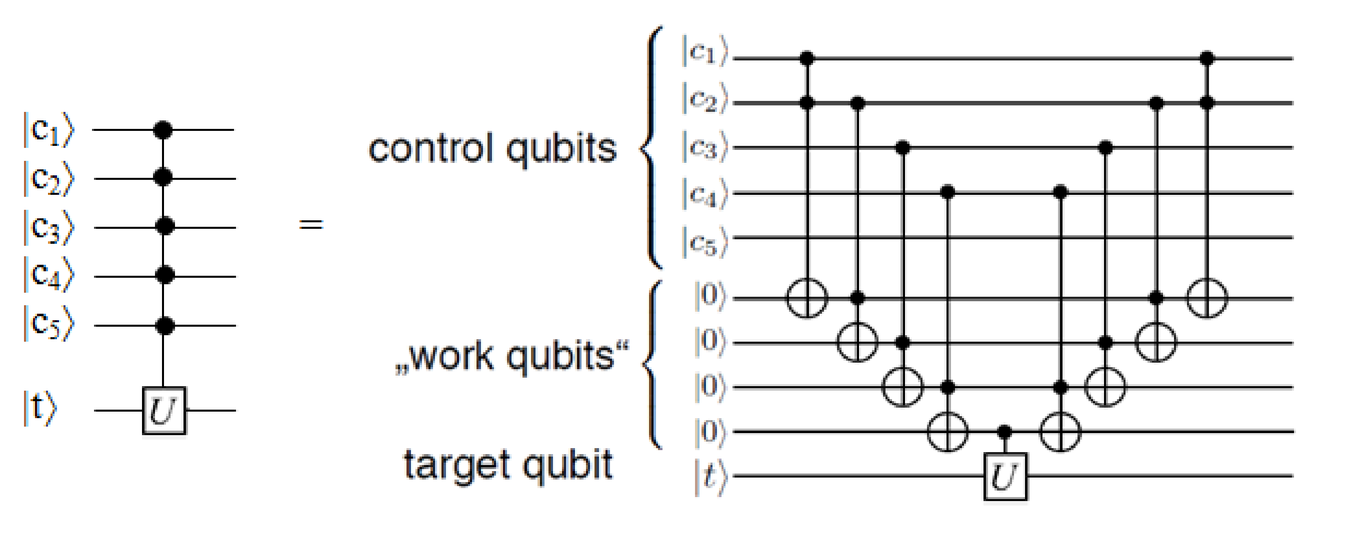
The shift operator for the quantum walk on the hipercube of dimension 4 :



The first gate flips the target qubit if and only if the control qubits are set to 0.

Discret time quantum walk on the hypercube

In the case of $n>4$, the quantum circuit for the shift operator uses (Controlled)^m-NOT gates. Such gates can be implemented using $2(m-1)$ CCNOT gates and $(m-1)$ ancilla qubits as follows



Discret time quantum walk on the hypercube

```
operator CmNOT(int m, qureg x, qureg y) {
  qureg a[m-1]; //ancilla qubits
  int i;
  CCNot(x[m-1], x[m-2], a[m-2]);
  for i=3 to m-1 {
    CCNot(x[m-i],a[m-i+1],a[m-i]);}
  for i=3 to m-1 step -1 {
    CCNot(x[m-i],a[m-i+1],a[m-i]);}
  CCNot(x[m-1], x[m-2], a[m-2]);
}
```

Conclusions

- In the last years new model of quantum algorithms have apperead: the quantum walk based algorithms.
- Quantum walks present different behaviour than classical random walks.
- In absence of quantum devices, quantum computing simulators helps programmers to understand the constraints imposed by these devices. I presented a quantum circuit for the shift operator of a quantum walk on the hypercube
- I presented a QCL implementation of the quantum walk algorithm on the hypercube .

Thank you!

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This is a preliminary version of the InfoTech-2015 Conference Program and some changes could be made. The participants will get the official Conference Program during the On-Site Registration.

International Conference InfoTech-2015
Thursday, 17th September 2015

13:30 – 14:00 (Hall 5)	Official Opening Session

Opening Speech and Introduction
Prof. Radi Romansky, D.Sc.
Chairperson of the Organizing Committee (Bulgaria)

Invited Keynote

14:00 – 15:00 (Hall 5)	Report Session

Chairpersons: *Prof. Radi Romansky, D. Sc. (Bulgaria)*
Prof. Plamen Mateev, Ph.D. (Bulgaria)

Section 'A': Information Technologies

Specialized Information Technologies

A01 **A Reference Point Genetic Algorithm for Multi-Criteria Job Shop Scheduling Problems**
Vassil Guliashki, Leonid Kirilov
Institute of Information and Communication Technologies – BAS (Bulgaria

A02 **Comparative Analysis of the Electoral Distribution Methods in Bulgarian Voting Legislation**
Iliya Goranov
Institute of Information and Communication Technologies – BAS (Bulgaria)

A03 **Design of Portable ECG Module**
Valentina Markova, Ventseslav Draganov, Edy Velikov, Yasen Kalinin
Technical University – Varna (Bulgaria)

15:00 – 15:40 (Foyer)	Coffee Discussion

15:40 – 18:00 (Hall 5)	**Report Session**

Chairpersons: *Prof. Vangel Fustik, Ph.D. (Rep. of Macedonia)*
Assoc. Prof. Dimitar Tsanev, Ph.D. (Bulgaria)

Section 'B': Information Security, Privacy and Network Applications

Cloud Computing

B03 **Large Scale Data Processing in the Cloud**
André Martin, Christof Fetzer
TU Dresden, Faculty of Computer Science, Dresden (Germany)

Web Applications

B04 **Web Applications Variability – Technological Trends and Models**
Iliya Nedyalkov [1], *Ivo Damyanov* [2]
[1] University of National and World Economy, Sofia (Bulgaria)
[2] South-West University, Blagoevgrad (Bulgaria)

Section 'C': Intelligent Systems and Applications

Intelligent and Agent Systems

C01 **Multi-Agent Framework for Intelligent Networks**
Georgi Tsochev [1], *Roumen Trifonov* [2], *Radoslav Yoshinov* [3]
[1] Technical University - Sofia (Bulgaria)
[2] Computer Systems Dept. at Technical University - Sofia (Bulgaria)
[3] Director of Telematics Laboratory at BA Sciences (Bulgaria)

C02 **Step by Step Data Preprocessing for Data Mining. A Case Study**
Mirela Danubianu
"Stefan cel Mare"University of Suceava (Romania)

Section 'D': Technologies for System Design

Computer Architectures and Automation of System Design and Research

D01 **Quantum Circuits for Quantum Walks on the Hypercube**
Adina Bărîlă
"Ștefan cel Mare" University of Suceava (Romania)

D02 **Daily Optimal Operation of Power Plants in a Complex Power System**
Sofija Nikolova-Poceva, Anton Causevski, Vangel Fustik
University "Ss. Cyril and Methodius", Skopje (Rep. of Macedonia)

20:00 – 23:00	**Official Conference Diner**

International Conference InfoTech-2015
Friday, 18th September 2015

09:00 – 11:20 (Hall 5) **Report Session**

Chairpersons: *Assoc. Prof. Roumen Trifonov, Ph.D. (Bulgaria)*
Assoc. Prof. Maria Nikolova, Ph.D. (Bulgaria)

Section 'A': Information Technologies

Actual Information Technologies

A04 **Griderages with Curvelinear Elements from Plane Circumference**
Liliya Petrova
Dep. "Mechanics", VTU "T. Kableshkov" (Bulgaria)

A05 **IT Project "Challenges 3D-the Island"**
Krasimir Bozhinov, Luchezar Ilieav, Ivan Dzhendov
Secondary School of Maths and Science "Prof. Asen Zlatarov" (Bulgaria)

A06 **GetYourStats**
Svetlin Yotov
SiS Develop (Bulgaria)

Section 'E': Technological Aspects of e-Governance and Data Protection

Technological Aspects of e-Governance

E01 **Perspectives for ICT Applications in e-Democracy**
Maria Nikolova
New Bulgarian University, Sofia (Bulgaria)

e-Learning and Educational Aspects

E04 **E-Learning Project for Interoperability in the Context of Electronic Government**
Milena Yorfanova [1], Roumen Trifonov [2], Slavcho Manolov [3]
[1] TEZA (Bulgaria)
[2] Department Computer Systems at Technical University - Sofia (Bulgaria)
[3] Chairman of the Board and CEO of Association EDIBUL (Bulgaria)

E05 **MOOCs and MOOC Platforms – Brief Survey, Innovations, Trends and Future**
Tatyana Ivanova
Technical University of Sofia, CEE (Bulgaria)

E06 **Game Strategies in Education Process**
Iglika Getova
University of Library Science and Information Technologies (Bulgaria)

11:20 – 12:00 (Foyer) Poster Session and Coffee Discussion

Chairperson: *Assoc. Prof. Irina Noninska, Ph.D. (Bulgaria)*

Section 'A': Information Technologies

Web Applications

A07 **Design and Develop a Webgis Application for Android**
P. Salla, S. Kolios, C. Stylios
Technological Educational Institute of Epirus (Greece)

Section 'B': Information Security, Privacy and Network Applications

Information Security and Privacy

B01 **Security and Privacy Principles Realization in e-Learning Architecture**
Radi Romansky, Irina Noninska
Technical University of Sofia (Bulgaria)

B02 **Formalization and Modelling of Secure Access in e-Learning Environment**
Radi Romansky, Irina Noninska
Technical University of Sofia (Bulgaria)

Section 'C': Intelligent Systems and Applications

Intelligent and Agent Systems

C03 **A Ridge Regression Approach for Quantum Machine Learning**
Vanya Markova, Ventseslav Shopov
Institute for SER – BAS (Bulgaria)

C04 **Approach for Quantum Clustering with Constrains**
Vanya Markova, Ventseslav Shopov
Institute for SER – BAS (Bulgaria)

C05 **Approach for Reducing the Number of Attributes in Feature Engineering**
Ventseslav Shopov, Vanya Markova
Institute for SER – BAS (Bulgaria)

C06 **Fast Adaptive Learning Algorithm for Classification of Time Series with Sigmoid Treshold**
Ventseslav Shopov, Vanya Markova, Velko Iltchev
Institute for SER – BAS (Bulgaria)

Knowledge-Based Applications

C07 **FPGA Robotic System for Tracking Objects and Digital Image Processing**
Rosen Spirov, Georgi Angelov
Technical University of Varna (Bulgaria)

Section 'D': Technologies for System Design

Computer Architectures and Automation of System Design and Research

D03 **Selection the Approximating Function for Isobologram Modeling**
Kaloyan Yankov
Trakia University (Bulgaria)

D04 **Implementation of Hardware and Software Modules for Lab Robots**
Andrei Hinkov, Mladen Milushev
Technical University of Sofia (Bulgaria)

D05 **Dependence of Three-Phase Distribution Transformer Core Losses From Current Harmonics**
Mihail Digalovski, Goran Rafajlovski, Krste Najdenkoski
University “Ss. Cyril and Methodius”, Skopje (Rep. of Macedonia)

Section ‘E’: Technological Aspects of e-Governance and Data Protection

Technological Aspects of e-Governance

E02 **Standartozation of Electronic Identity Management**
Slavcho Manolov [1], Roumen Trifonov [2], Radoslav Yoshinov [3]
[1] Chairman of the Board and CEO of Association EDIBUL (Bulgaria)
[2] Department Computer Systems at Technical University - Sofia (Bulgaria)
[3] Director of Telematics Laboratory at BAS (Bulgaria)

E03 **E-Government Applications for integrated Access to Complex Data Resources Using Multi-Agent Systems**
Roumen Trifonov [1], Slavcho Manolov [2], Radoslav Yoshinov [3]
[1] Department Computer Systems at Technical University - Sofia (Bulgaria)
[2] Chairman of the Board and CEO of Association EDIBUL (Bulgaria)
[3] Director of Telematics Laboratory at BAS (Bulgaria)

12:00 (Foyer) **Conference InfoTech-2015 Closing**

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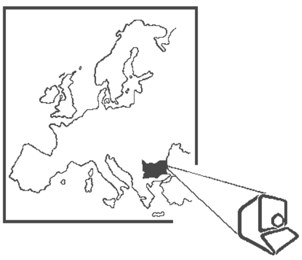

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